Simplification of High Frame Rate Imaging System with Coordinate Rotation

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Abstract **– A method to greatly simplify the high-frame-rate (HFR) imaging system using a rotation of coordinates in image reconstruction was developed. A theory of Fourier image reconstruction was also developed and both *in vitro* (on an ATS539 tissue-mimicking phantom) and *in vivo* (on a human heart) experiments were performed to verify the theory.**

Keywords - limited diffraction beams; high frame rate; HFR; medical imaging; beamforming

I. INTRODUCTION

Based on the limited-diffraction beam theory [1]-[4], a high-frame-rate (HFR) imaging method for two-dimensional (2D) and three-dimensional (3D) imaging was developed in 1997 [5]-[9] and has been extended recently to explicitly include limited-diffraction array beam and steered plane wave transmissions to increase image field of view and to achieve equivalent dynamic focusing in both transmission and reception [10]-[19]. This method has the advantage that it could be implemented with simpler hardware than the conventional delay-and-sum (D&S) method [20] because the limited-diffraction array beam transmissions [21]-[23] can be approximated with square-wave aperture weightings [10] and the fast Fourier transform (FFT) can be used to reconstruct images. The method can also use multiple steered plane waves [5], [24]-[25].

In this paper, the HFR imaging method is further simplified using rotated Cartesian coordinates in which one of the rotated axes coincides with the transmission wave vector that corresponds to the central frequency of a 2D array transducer. A new mathematical relationship between the Fourier transform of a 3D object function and the limited-diffraction array beam weightings or 2D Fourier transform over the transducer aperture is developed. Images are reconstructed for various transmission wave vectors, rotated back, and then summed coherently (to obtain a high resolution, contrast, and large field of view) in the original coordinates to form the final image at a high frame rate. The summation could also be done incoherently to reduce speckles. With this method, the unnecessary high spatial-frequency components, which are produced by the off-axis transmit wave vectors, of the echo signals over the transducer aperture are greatly reduced. This allows the use of a larger sampling interval in the Fourier space, reducing the number of points in the fast Fourier transform (FFT) and the memory usage. In addition, a depth-dependent spatial filter can be more easily applied to increase signal-to-noise ratio by matching the filter bandwidth with the transverse spatial bandwidth of echo signals over depths (i.e., smaller spatial bandwidth at larger depths).

This work was supported in part by a grant, HL60301, from the National Institutes of Health of USA.

To verify the method, a home-made general-purpose high-frame rate imaging system [10], [26]-[27] was used to acquire radiofrequency (RF) echo signals from an ATS539 tissue-mimicking phantom (ATS Laboratory, Inc) and a human heart. In the experiments, a 2.5-MHz center frequency, 128-element, and 19.2-mm aperture broadband phased-array transducer was used to obtain 2D images over a +/-45-degree field of view. The echo signals were digitized to 12 bits at 40 MHz. Results show that the method can greatly reduce the number of points required in the FFT operations while maintaining the quality of the reconstructed images.

II. THEORY

The details of the theory of the HFR imaging method are given in [10] and thus will not be repeated here. With the X-wave formulas [1]-[4], one obtains a relationship between the Fourier transform of an object function and the RF echo signals as follows [10]:

$$\begin{aligned}\tilde{R}_{k_{x_R}+k_{x_T},k_{y_R}+k_{y_T}}(\omega) &= \frac{A(k)T(k)H(k)}{c^2}\int_V f(\vec{r}_0)e^{i(\vec{K}^R+\vec{K}^T)\cdot\vec{r}_0}d\vec{r}_0 \\ &= \frac{A(k)T(k)H(k)}{c^2}F(\vec{K}^R+\vec{K}^T) \\ &= \frac{A(k)T(k)H(k)}{c^2}F'\left[(\vec{K}^R+\vec{K}^T)\Theta\right] \\ &= \frac{A(k)T(k)H(k)}{c^2}F'(k_{x_{R_2}},k_{y_{R_2}},k_{z_{R_2}}+k), \quad (1)\end{aligned}$$

where $\tilde{R}_{k_{x_R}+k_{x_T},k_{y_R}+k_{y_T}}(\omega)$ is the Fourier transform of the time-varying RF echo signal when the transducer receive aperture is weighted at the spatial frequencies k_{x_R} and k_{y_R}, $\vec{r}_0=[x_0,y_0,z_0]^t$ and $\vec{r}_2=[x_2,y_2,z_2]^t$ are the original and rotated coordinates, respectively (rotating angles are ζ_T and θ_T in Fig. 1), the superscript, t, means a transpose of a vector or matrix, $H(\omega/c)=\{1,\ \omega\geq 0;\ 0,\ \omega<0\}$ is the Heaviside step function [28], $A(k)$ and $T(k)$ are the transmit and receive transfer functions, respectively [29], $k=\omega/c$ is the wave number, where $\omega=2\pi f$ is the angular frequency, and f is the temporal frequency, c is the speed of sound, $f(\cdot)$ and $F(\cdot)$ are a 3D object function and its Fourier transform, respectively, V is the volume of the object. $F'(\cdot)$ is the Fourier transform of $f(\cdot)$ at the rotated coordinates.

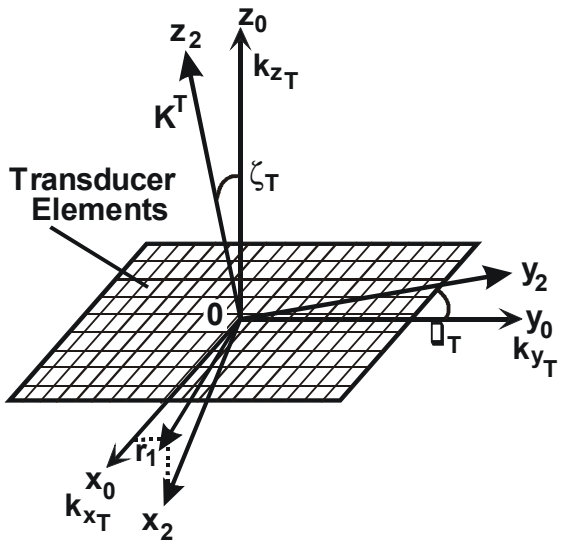

Figure 1. Coordinates before ($\vec{r}_0$) and after rotation ($\vec{r}_2$).

The transmission and reception wave vectors are given by:

$\vec{K}^T = (k_{x_T}, k_{y_T}, k_{z_T})$ and $\vec{K}^R = (k_{x_R}, k_{y_R}, k_{z_R})$,

respectively, where $k_{z_T} = \sqrt{k^2 - k_{x_T}^2 - k_{y_T}^2}$ and $k_{z_R} = \sqrt{k^2 - k_{x_R}^2 - k_{y_R}^2}$.

The rotation matrix and its inversion are given by:

$$\Theta = \begin{bmatrix} \cos\zeta_T \cos\theta_T & -\sin\theta_T & \sin\zeta_T \cos\theta_T \\ \cos\zeta_T \sin\theta_T & \cos\theta_T & \sin\zeta_T \sin\theta_T \\ -\sin\zeta_T & 0 & \cos\zeta_T \end{bmatrix} \quad (2)$$

and

$$\Theta^{-1} = \begin{bmatrix} \cos\zeta_T \cos\theta_T & \cos\zeta_T \sin\theta_T & -\sin\zeta_T \\ -\sin\theta_T & \cos\theta_T & 0 \\ \sin\zeta_T \cos\theta_T & \sin\zeta_T \sin\theta_T & \cos\zeta_T \end{bmatrix} = \Theta^t \quad , \quad (3)$$

respectively.

If a point in the coordinates before and after the rotation is given by $\vec{r}_0 = x_0\vec{i}_0 + y_0\vec{j}_0 + z_0\vec{k}_0$ and $\vec{r}_2 = x_2\vec{i}_2 + y_2\vec{j}_2 + z_2\vec{k}_2$, respectively, one has: $\vec{r}_0 = \Theta\vec{r}_2$ and $\vec{r}_2 = \Theta^{-1}\vec{r}_0$. In addition, one obtains: $\vec{K}_2^T = (k_{x_{T_2}}, k_{y_{T_2}}, k_{z_{T_2}}) = \vec{K}^T\Theta = (0,0,k)$ and $\vec{K}_2^R = (k_{x_{R_2}}, k_{y_{R_2}}, k_{z_{R_2}}) = \vec{K}^R\Theta$ for the transmit and receive wave vectors in the rotated coordinates (assuming the transmit wave vector is aligned with a rotated axis), respectively, where $k_{z_{T_2}} = \sqrt{k^2 - k_{x_{T_2}}^2 - k_{y_{T_2}}^2}$ and $k_{z_{R_2}} = \sqrt{k^2 - k_{x_{R_2}}^2 - k_{y_{R_2}}^2}$.

From the formulas above (Eq. (1)), it is clear that the relationship between the Fourier transform, $F'(k_{x_{R_2}}, k_{y_{R_2}}, k_{z_{R_2}} + k)$, of an object function, $f(\vec{r}_0)$, and the Fourier transform of received RF echo signals (can be easily rotated by linear phase delays) in the rotated coordinates is exactly the same as Equation (15) of [5], which is simpler to implement than that in the original coordinates.

III. Experiments and Results

To verify the method and the formulas above, both *in vivo* and *in vitro* experiments were performed with a home-made HFR imaging system [10], [26]-[27]. The system has 128-channels to drive a 2.5-MHz broadband phase array transducer of 128 elements. RF echoes were digitized to 12 bits at 40 MHz and were used for image reconstructions.

Fig. 2 is a reconstructed strip of image of an ATS539 tissue-mimicking phantom with a single transmission steered at -45º. After a rotation of the echo signals, the strip of image reconstructed with Eq. (1) is steered at 0º in the rotated coordinates (see Fig. 3). It is clear that the images reconstructed before and after the rotation are similar.

Figs. 4 and 5 are images reconstructed with and without the rotation of coordinates for an ATS539 tissue-mimicking phantom. Figs. 6 and 7 are images reconstructed with and without the rotation of coordinates for a human heart *in vivo*. The image frame rate is about 486 per second (187 μs between transmissions). The sizes of images in Figs. 2-7 are 153.6 mm in width and 120 mm in depth.

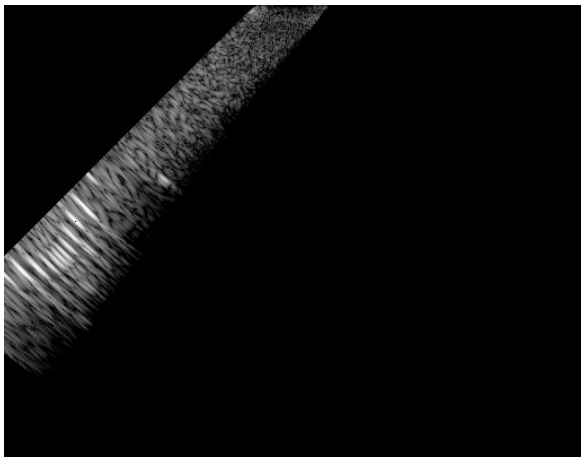

Figure 2. A strip of reconstructed iamge of an ATS539 tissue-mimicking phantom with one transmission steered at -45º without coordinate rotation using Eq. (1) (see Reference [10] for details of image reconstructions).

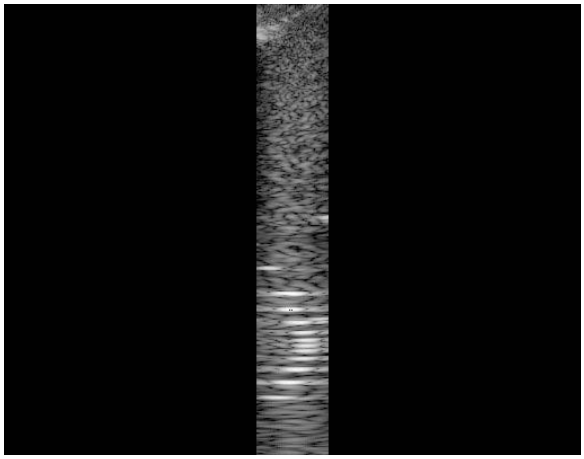

Figure 3. A strip of reconstructed iamge of an ATS539 tissue-mimicking phantom with one transmission steered at -45º with coordinate rotation using Eq. (1) (see Reference [10] for details of image reconstructions).

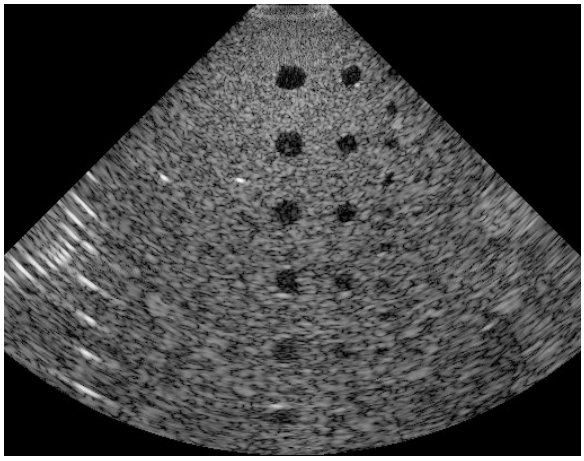

Figure 4. Reconstructed iamge of an ATS539 tissue-mimicking phantom with 11 transmissions steered from -45º to 45º without coordinate rotation using Eq. (1) (see Reference [10] for details of image reconstructions).

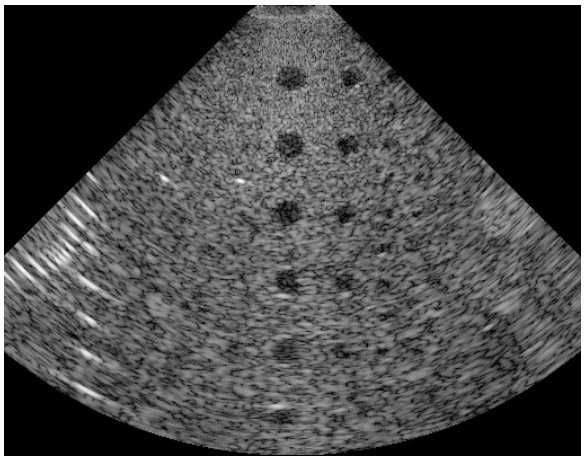

Figure 5. Reconstructed iamge of an ATS539 tissue-mimicking phantom with 11 transmissions steered from -45º to 45º with coordinate rotation using Eq. (1) (see Reference [10] for details of image reconstructions).

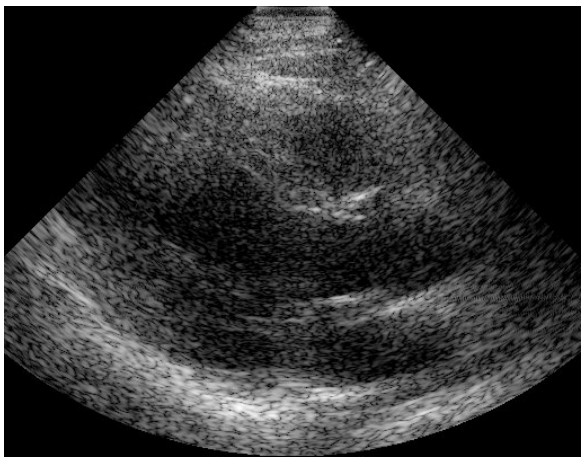

Figure 6. Reconstructed iamge of a human heart *in vivo* with 11 transmissions steered from -45º to 45º without coordinate rotation using Eq. (1) (see Reference [10] for details of image reconstructions).

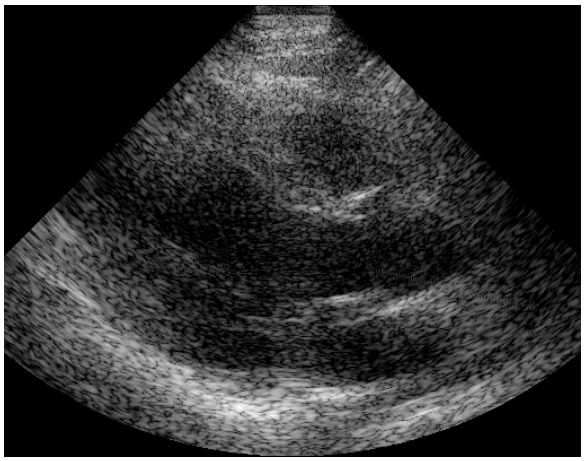

Figure 7. Reconstructed iamge of a human heart *in vivo* with 11 transmissions steered from -45º to 45º with coordinate rotation using Eq. (1) (see Reference [10] for details of image reconstructions).

IV. Conclusion

The method developed using a rotation of coordinates may be useful to greatly reduce the complexity of imaging systems for reconstructing high-frame-rate images.

Acknowledgments

This work was supported in part by a grant HL 60301 from the National Institutes of Health.

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